COSC201: Tutorial Week 2
Proofs: Induction

Use induction to prove the following[1]:

1. Prove that $2n + 1 = O(2^n)$.
2. Prove that $n^2 = O(n!)$.
3. Prove that $1^2 + 2^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$ for all $n \geq 1$.
4. Prove that $4^n - 1$ is divisible by 3 for all $n \geq 1$.
5. Show that $1^2 + 2^2 + \ldots + n^2 = n(n + 1)(2n + 1)/6$ for all $n \geq 1$.
6. Prove that $1^3 + 2^3 + \ldots + n^3 = [n(n + 1)/2]^2$ for all $n \geq 0$.
7. Prove that $n^3 - 7n + 3$ is divisible by 3 for all $n \geq 0$.
8. Prove that $1 \cdot 2 + 2 \cdot 3 + \ldots + (n - 1) \cdot n = n(n - 1)(n + 1)/3$ for all $n \geq 2$.
9. *Extension exercise:* Prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for all $n \geq 0$.

[1] Note: there are more tutorial questions here that can reasonably be completed in the tutorial. The extra questions are for practice.