COSC201: Tutorial Week 4 Proofs: Induction

Use induction to prove the following¹:

- 1. Prove that $2n + 1 = O(2^n)$.
- 2. Prove that $n^2 = O(n!)$.
- 3. Prove that $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for all $n \ge 1$.
- 4. Prove that $4^n 1$ is divisible by 3 for all $n \ge 1$.
- 5. Show that $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for all $n \ge 1$.
- 6. Prove that $1^3 + 2^3 + \ldots + n^3 = [n(n+1)/2]^2$ for all $n \ge 0$.
- 7. Prove that $n^3 7n + 3$ is divisible by 3 for all $n \ge 0$.
- 8. Prove that $1 \cdot 2 + 2 \cdot 3 + \ldots + (n-1) \cdot n = n(n-1)(n+1)/3$ for all $n \ge 2$.
- 9. Extension exercise: Prove that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for all $n \ge 0$.

¹Note: there are more tutorial questions here that can reasonably be completed in the tutorial. The extra questions are for practice.