Cosc 201
Algorithms and Data Structures
Lecture 15 (14/4/2025)
Operations on binary search trees

Brendan McCane
brendan.mccane@otago.ac.nz
And
Michael Albert

Binary search tree definition

A binary search tree (BST) is a collection of nodes with one distinguished node called the root

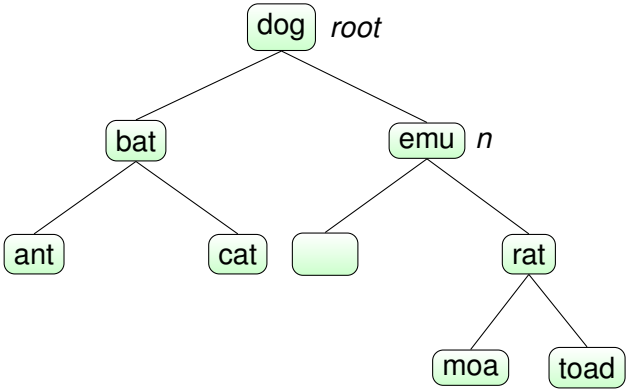
- A node contains data called a *key* which comes from some ordered type (e.g., `String`).
- Each node can have at most two children - there are two fixed slots called *left child* and *right child*.
- The left child of a node and all their descendants are called the left subtree at the node. The right subtree is defined similarly.
- The key values at all nodes in the left subtree of a node must be less than the key of the node.
- The key value at all nodes in the right subtree of a node must be greater than the key of the node.
- Duplicate keys are not permitted.

Notation for BST pseudo-code

If n is a node:

- $n.key$ for its key,
- $n.l$ and $n.r$ for its left and right children
- $n.p$ for its parent
- **nil** for a "not here" marker, e.g., $n.r =$ **nil** means "n has no right child".

Example BST



- $root.p = \textbf{nil}$
- $root.r = n$
- $n.p = root$
- $n.key = emu$
- $n.l = \textbf{nil}$
- $n.r.key = rat$

Searching in a BST

We want to search in a BST for a key k returning **true** if it is found and **false** if it is not.

```
1:  n ← root
2:  while n ≠ nil do
3:      if n.key = k then
4:          return true
5:      else if n.key < k then
6:          n ← n.r
7:      else
8:          n ← n.l
9:      end if
10: end while
11: return false
```

Searching in a BST, recursively

We want to search in a BST for a key k returning **true** if it is found and **false** if it is not.

The method *search*(k) simply calls *search*(k, *root*) where the two-parameter version, *search*(k, n) is defined by:

```
1: if n = nil then
2:     return false
3: else if n.key = k then
4:     return true
5: else if n.key < k then
6:     return search(k, n.right)
7: else
8:     return search(k, n.left)
9: end if
```

Adding to a BST

We want to add a key k to a BST returning **true** if it is added and **false** if it was already present.

```
1:  p ← nil, c ← root
2:  while c ≠ nil do
3:      p ← c
4:      if p.key = k then
5:          return false
6:      else if p.key < k then
7:          c ← p.r
8:      else
9:          c ← p.l
10:     end if
11: end while
12: make a child node of p with value k
13: return true
```

Making a child

We want to add a child node with key k to a parent node p.

Assumption This is **really** what we want to do, i.e., k is not the key of p and the position where this node will be added is currently **nil**.

1: $c \leftarrow$ **new**(k), $c.p \leftarrow p$
2: **if** $p.key < k$ **then**
3: $\quad p.r \leftarrow c$
4: **else**
5: $\quad p.l \leftarrow c$
6: **end if**

Removing from a BST

This one is a little complicated! There are some cases to consider.

- First find the node, c, containing the key to be removed and its parent node p.
- If c has no children then just replace it by **nil** as the (appropriate) child of its parent.
- If c has only one child, d, replace it by d as the (appropriate) child of its parent.
- But, if c has two children ...
- A new idea – find the *successor* of c. That's a node with at most one child. Delete it (previous case) and replace c's key by the successor's key.

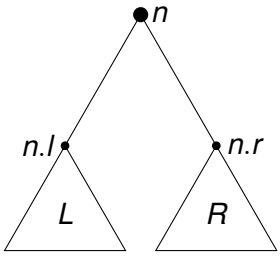
What's the successor?

The *successor* of a node n in a BST is the leftmost descendant of its right child.

Assumption The node n has a right child which is not **nil**

1: $c \leftarrow n.r$
2: **while** $c.l \neq$ **nil do**
3: $\quad c \leftarrow c.l$
4: **end while**
5: **return** c.

Traversing a BST



There are three different traversals we commonly consider in a BST. They are all based on the picture to the left.

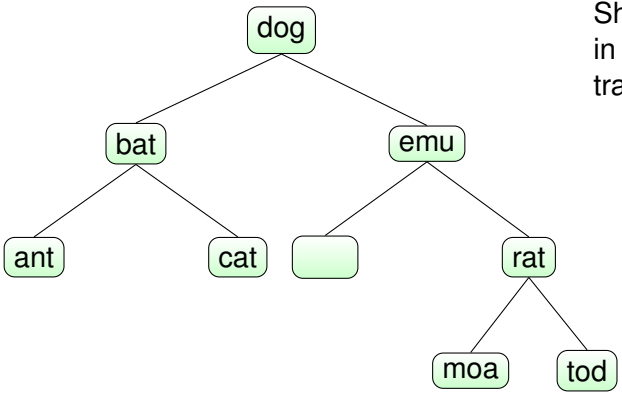
We need to "visit" n, visit all the nodes in L and visit all the nodes in R. The three different traversals represent three different choices of what order to make those visits in.

Preorder
Visit n
Traverse L
Traverse R

Inorder
Traverse L
Visit n
Traverse R

Postorder
Traverse L
Traverse R
Visit n

Traversal example



Showing the keys of the nodes in the order visited by the three traversals:

Preorder	dog, bat, ant, cat, emu, rat, moa, tod.
Inorder	ant, bat, cat, dog, emu, moa, rat, tod.
Postorder	ant, cat, bat, moa, tod, rat, emu, dog.

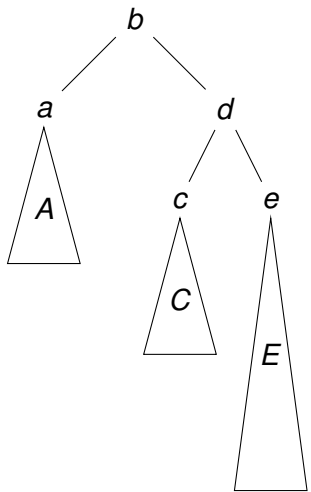
Long branches are a problem

- The performance bounds for all of the BST operations are linear in the length of the longest branch.
- Ideally, we'd like our BST's to look more like heaps (wide and shallow) than long spindly branches.
- Unlike heaps though, the structure of a BST is out of our control – it depends on the order in which elements are added.
- In particular if they are added in their natural order we'll get one great big long branch and the BST performance will degrade to that of a linked list.
- A BST in which there are no "long" branches is called *balanced*.
- So, are there mechanisms by which we can ensure balance? Can they be implemented with limited additional overhead?

Is balancing even possible?

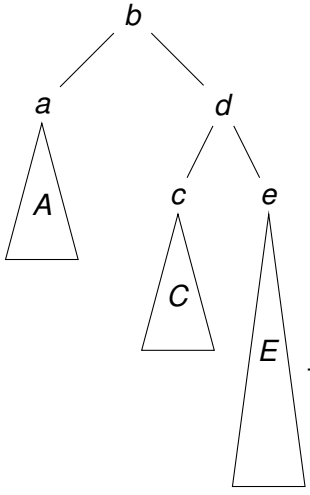
- In a global way, yes.
  - Perform an in-order traversal of the whole tree and save the results to a (sorted) array.
  - Now repeatedly bisect the array - use the middle element as the root
  - Recursively build the left and right subtrees from the part before the middle and the part after.
  - But, this hardly meets the “limited additional overhead” criterion!
- We need some local operation that helps to restore balance.
- The name of this operation is rotation.
- So, what’s that?

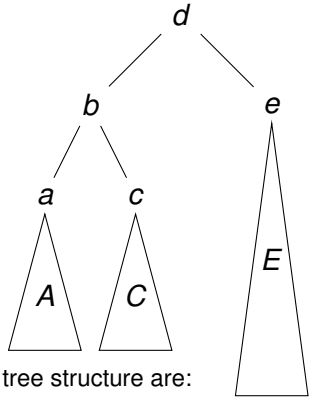
The bad situation (and its fix)



- Suppose that in this BST, there is a longer chain in E than elsewhere.
- The idea is to rotate d upwards (to become in this view at least the root).
- Which gives . . .
- And that's an improvement.

Side by side





The internal changes to the tree structure are:

- Change *b*'s right child to point to *c*.
- Change *d*'s left child to point to *b*.
- Change the link on *b*'s parent (not shown) that pointed to *b* to point to *d* instead.